is the result of extensive analyses carried out by the authors and others. Devoted, as it is, primarily to special-purpose network analyzers, its audience is a somewhat limited one. Nonetheless, the authors are to be commended for their contributions to a complex subject, and for their constant attention to the theoretical as well as to the practical aspects of their analogues.

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**45[P, Z].**—HELLMUT GOLDE, *Fortran* II *and* IV *for Engineers and Scientists*, The Macmillan Company, New York, 1966, xvi + 224 pp., 28 cm. Price $4.50.

"Primarily a text for beginning students in engineering and science on the college level," this book represents, with a few exceptions, a complete treatment of the subject matter in a clear and lucid style. After the usual introductory material, the elements of the FORTRAN language are presented in Chapters 3–9. The dialect is that of level one of the proposed American Standard for FORTRAN. The FORTRAN IV extension is discussed in Chapter 10. A presentation of the elements of the language is motivated by means of one or more coding problem(s) in each chapter. Examples of correct and incorrect coding are given throughout the text. Basic numerical problems associated with fixed precision floating point quantities and the necessary programming to avoid these difficulties are also discussed. Complete statistics are available concerned with the characteristics of the compilers for various computers (appendices A and B).

Although the overall treatment of the subject matter is good, there are a few weaknesses. Scanty material is presented on the generation and review of binary information stored peripherally, an important aspect of many large scientific problems. A discussion of the computed GØ TØ statement and EQUIVALENCE statement is left for the concluding Chapter 11. Thus, the frequent use of these statements, which occurs in everyday situations, is not reflected in the coding problems of the text. A glossary of terms is not included.

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**46[S].**—B. L. MOISEIWITSCH, *Variational Principles*, John Wiley & Sons, New York, 1966, x + 310 pp., 24 cm. Price $14.00.

Variational principles have long played two major roles in mathematical physics; one as great unifying principles through which the different equations can be expressed in elegantly simple form, and the other as remarkably useful computational tools for the accurate determination of discrete eigenvalues such as the vibration frequencies of classical systems and the bound state energies of quantum mechanical systems. In the latter role, variational principles represent a small triumph of man over nature. The fractional error in the quantity to be determined, the "output," is proportional to the square of the fractional error in the "input" information, almost giving one the eerie feeling that some law of thermodynamics is being violated; the "input" information is represented, for example, by a guess at the shape of a vibrating string. Furthermore, the sign of the "output" error is known.

A number of exciting developments in the field of variational principles have taken place in the past 20 years, particularly with regard to the analysis of scattering problems. (In the quantum mechanical case, the concern is then with the continuous portion of the energy spectrum and the problem is to determine not the energy but quantities such as phase shifts which determine the scattering.) The present time is therefore appropriate for a good review of the subject. *Variational Principles* provides just that.

The introductory material, on the problem of expressing the equations of motion of classical mechanics in the form of variational principles, as, for example, the principle of least action, is, almost of necessity, primarily a standard treatment. The author then turns to the basic variational principle of optics, Fermat's principle of least time, and to the establishment of an analogue of Hamilton's principle which provides the quantum dynamical equations of motion in a Lagrangian form. The latter material, the work of Schwinger, is not yet standard textbook fare; the treatment is good but too terse.

The variational principle formulation of the field equations of physics is enriched by a variety of applications from electromagnetism, sound waves, etc., that should be very helpful to the student of mathematical physics. One might hope that in any later edition Moiseiwitsch will also include examples from magnetohydrodynamics, nonequilibrium thermodynamics, and other topics not often studied by graduate students of physics or mathematics. It is my impression that developments in variational principles in one field have taken unconscionably long to spread to other fields even when they were essentially immediately applicable. The rate of flow of information from one area to another can be increased by the proper selection of examples from the different areas.

Many detailed examples of the determination of discrete eigenvalues are covered, a number from atomic physics, where the "input" is a trial function. A recent development of potentially great significance, a variational principle for an arbitrary operator, or rather for matrix elements of the operator, is discussed.

The last third of the book is devoted to the use of variational principles in scattering theory. This much space is fully justified by the developments in the field, and is desirable since the author has been among the most active workers in the field.

Variational principles are established for scattering phase shifts and scattering amplitudes, and a number of examples, largely from atomic physics, are considered in sufficient detail to be highly educational. The original formulation of scattering theory provided a variational principle which was weaker than its discrete energy eigenvalue analogue; it provided a second order error but not the sign of the error and therefore did not provide a bound. (In the space of the parameters used in the trial function, there might be a saddle point rather than an absolute extremum.) More recently the full analogue of the Rayleigh-Ritz variational bound on discrete eigenvalues was developed. The treatment in the book is limited to that incident energy, zero, for which the variational bound formulation of scattering theory assumes its simplest form. Variational bounds are studied in a number of important and by no means trivial examples, including that of the zero energy scattering of electrons by hydrogen atoms.

The book concludes with a treatment of the basic work by Lippmann and Schwinger on formal time-dependent scattering theory, but, for only the second instance in the volume, the treatment is probably too terse to be really useful. In general, the treatment of material throughout the text is sufficiently thorough to enable second year graduate students of physics not only to follow but to profit considerably; with the possible exception of some of the formal material on quantum mechanics and the treatment of the Dirac equation, the same should be true for students of mathematics.

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**47[S, X].**—Harry H. Denman, Wilfried Heller & William J. Pangonis, *Angular Scattering Functions for Spheres*, Wayne State University Press, Detroit, Michigan, 1966, xix + 294 pp., 24 cm. Price \$7.50.

Let

$$i_\perp = \left| \sum_{n=1}^{\infty} \{A_n \pi_n (\cos \alpha) + B_n \tau_n (\cos \alpha)\} \right|^2,$$

$$i_\parallel = \left| \sum_{n=1}^{\infty} \{A_n \tau_n (\cos \alpha) + B_n \pi_n (\cos \alpha)\} \right|^2.$$

$\pi_n(x) = dP_n(x)/dx$, $\tau_n(x) = x\pi_n(x) - (1 - x^2)d\pi_n(x)/dx$, where $P_n(x)$ is the Legendre polynomial of degree $n$. Also the coefficients $A_n$ and $B_n$ depend on the Riccati-Bessel functions.

$S_n(x) = (\pi x/2)^{1/2} J_{n+1/2}(x)$, $C_n(x) = (-1)^n (\pi x/2)^{1/2} J_{-n-1/2}(x)$. $A_n$ and $B_n$ are functions of $S_n(\alpha)$ and $S_n(\beta)$ where $\beta = m\alpha$. This volume tabulates $i_\perp/\alpha^3$ and $i_\parallel/\alpha^3$ to 5S for $\alpha = 0.2(0.2)25$, $\alpha = 0°(5°)180°$, $m = 1.05(0.05)1.30$, 1.333.

The method of computation and the checks used are explained in detail, and the authors conclude that "the fifth figure in these tables is *correct* in most cases, and is *significant* almost always." (The italics are theirs.) The present tables are the most complete on the subject. For a description of the physical aspects of the problem to which the tables relate and previous tables, see *MTAC*, v. 3, 1949, p. 483–484 and *MTAC*, v. 6, 1952, p. 95–97.

Y. L. L.

**48[W, X].**—V. S. Nemchinov, Editor, *The Use of Mathematics in Economics*, The M. I. T. Press, Cambridge, Massachusetts, 1965, xxi + 377 pp., 26 cm. Price \$12.50.

The present work constitutes a sample of recent East European (principally Soviet) work on mathematical economics.

The first article, by V. S. Nemchinov, gives an introductory discussion of industrial input-output matrices and their applications, with emphasis on planning of uniform growth. A gross national balance sheet for the Soviet economy (years